Extended Dictionary Learning : Convolutional and Multiple Feature Spaces

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• Sparse Coding for Image Processing Applications

• Coupled Dictionary Training

• Learning Deep Features



Sparse Representations (SR) Framework



Sparse Signal Modeling



Approximation: Greedy vs. Relaxation Techniques



Non-linear Sparsity Models



- Quantization^[1] $\rightarrow \mathcal{F}(\mathbf{y}, \mathbf{s}) = \mathcal{Q}(\mathbf{y}, \mathbf{s}) \implies \min \|\mathbf{y} \mathcal{Q}(\mathbf{\Phi}\mathbf{D}\mathbf{x})\|_2 \text{ s.t. } \|\mathbf{x}\|_0 \le K$
- Multipath SC
- Convolutional SC

[1] Recovery of quantized compressed sensing measurements, G Tsagkatakis, P Tsakalides, IS&T/SPIE EI 2015



Multipath sparse coding

- **Task:** *learn & combine recursive sparse coding through many pathways on multiple bags of patches of varying size to learn characteristic features*
- □ Application: Image Classification
- □ Algorithm:
 - Multipath Hierarchical Matching Pursuit
 - Mutual Incoherence Dictionary Learning



Bo, Liefeng, Xiaofeng Ren, and Dieter Fox. "Multipath sparse coding using hierarchical matching pursuit." *Computer Vision and Pattern Recognition (CVPR), 2013 IEEE Conference on.* IEEE, 2013.



Patch Based vs Convolutional Sparse Modeling



Convolutional Sparse Modeling

$$\underset{\mathbf{d},\mathbf{z}}{\operatorname{arg\,min}} \quad \frac{1}{2} ||\mathbf{x} - \sum_{k=1}^{K} \mathbf{d}_k * \mathbf{z}_k||_2^2 + \beta \sum_{k=1}^{K} ||\mathbf{z}_k||_1$$
subject to
$$||\mathbf{d}_k||_2^2 \leq 1 \text{ for } k = 1 \dots K.$$

Convolution operator

 ✓ Models local structures that appear anywhere in the image

- ✓ Translation Invariance
 - Orientations
 - Frequencies



Convolutional Sparse Coding

Optimization

- ADMM,
- Proximal Gradient,
- Block-Toeplitz

Applications

- Compression
- Super-resolution
- Inpainting
- HDR synthesis
- Deconvolution

$$\underset{\mathbf{d},\mathbf{z}}{\operatorname{arg\,min}} \qquad \frac{1}{2} ||\mathbf{x} - \sum_{k=1}^{K} \mathbf{d}_k * \mathbf{z}_k||_2^2 + \beta \sum_{k=1}^{K} ||\mathbf{z}_k||_1$$

subject to
$$||\mathbf{d}_k||_2^2 \le 1 \text{ for } k = 1 \dots K.$$



Hilton, and Lucey. "Optimization methods for convolutional sparse coding." *arXiv preprint 2014*. Felix, Heidrich, and Wetzstein. "Fast and Flexible Convolutional Sparse Coding." *IEEE CVPR 2015*



Convolutional Sparse Coding: ADMM Optimization

Initial Optimization Problem

$$\underset{\mathbf{d},\mathbf{z}}{\operatorname{arg\,min}} \quad \frac{1}{2} ||\mathbf{x} - \sum_{k=1}^{K} \mathbf{d}_{k} * \mathbf{z}_{k}||_{2}^{2} + \beta \sum_{k=1}^{K} ||\mathbf{z}_{k}||_{1}$$

subject to $||\mathbf{d}_{k}||_{2}^{2} \leq 1 \text{ for } k = 1 \dots K.$

General Formulation

$$\underset{\mathbf{d},\mathbf{z}}{\operatorname{argmin}} \ \frac{1}{2} \|\mathbf{x} - \mathbf{M} \sum_{k=1}^{K} \mathbf{d}_k * \mathbf{z}_k \|_2^2 + \beta \sum_{k=1}^{K} \|\mathbf{z}_k\|_1$$

subject to $\|\mathbf{d}_k\|_2^2 \le 1 \quad \forall k \in \{1, \dots, K\}$

M: diagonal matrix that masks out the boundaries of the padded estimation

$$\mathbf{n} \qquad (\mathbf{M}^T \mathbf{M} + \mathbb{I})\mathbf{x} = \mathbf{n}$$

Include the constraints in the objective function $\operatorname{argmin}_{\underline{1}}^{\underline{1}} \|\mathbf{x} - \mathbf{M} \sum \mathbf{d}_k \ast \mathbf{z}_k \|_2^2 + \beta \sum \|\mathbf{z}_k\|_1 + \sum \operatorname{ind}_C(\mathbf{d}_k),$

$$(\mathbf{M}^T\mathbf{M} + \mathbb{I})\mathbf{x} = \mathbf{b}$$

Auxiliary form

 $\mathbf{d}.\mathbf{z}$

where:

subject to Ay = z

$$\underset{\mathbf{d},\mathbf{z}}{\operatorname{argmin}} f_1(\mathbf{D}\mathbf{z}) + \sum_{k=1}^{K} \left(f_2(\mathbf{z}_k) + f_3(\mathbf{d}_k) \right) \quad f_1(\mathbf{v}) = \frac{1}{2} \|\mathbf{x} - \mathbf{M}\mathbf{v}\|_2^2, f_2(\mathbf{v}) = \beta \|\mathbf{v}\|_1, f_3(\mathbf{v}) = \operatorname{ind}_C(\mathbf{v})$$

ADMM formulation \rightarrow **DEDALE Tutorial Dav**

Paris, November 2016

argmin $h(\mathbf{y}) + g(\mathbf{z})$

Application: Convolutional Sparse Coding for Image Super-Resolution



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How to select the proper dictionaries



State-of-the-art: K-SVD



Dictionary Training: The K-SVD algorithm^[1]

Step 1 - Sparse Coding Stage: Find the best representation coefficient matrix X.

$$\min_{D,X} ||Y - DX||_F^2 \quad s.t \quad \forall i \quad ||x_i||_0 \le L, i \in (1, N)$$

$$\lim_{x_i} ||y_i - Dx_i||_2^2 \quad , s.t. \quad ||x||_0 \le L$$

Step 2- Dictionary Update Stage: Update one column at a time. For each column k={1,...,M}

Solve:

Representation error: $E_{k} = Y - \sum_{j}^{M} d_{j} x_{j}^{T}$

$$\min_{d_k, x_k} ||E_k - d_k x_k^T||_F^2 \qquad E_k = Y -$$

Define the group of indices that use the atom d_k:

 $\omega_k = \{i | 1 \le i \le M, x_k^T(i) \ne 0\}$

- Restrict E_k by choosing columns that correspond to ω_k
- SVD decomposition: $E_k = U\Delta V^T$. The first column of U is the updated column d_k . Update the first column of V to be the updated coefficients x_R^k

end

[1] M. Aharon et al: *K-SVD : An algorithm for designing over-complete dictionaries for sparse representation. IEEE Transactions on Image Processing. 2006*





Motivation for Joint Dictionaries





Super-resolution

Snapshot HDR





De-nighting DEDALE Tutorial Day Paris, November 2016

Spectral Super-Resolution



Coupled Dictionary Learning



Alternating Direction Method of Multipliers (ADMM) for Coupled Dictionary Learning (CDL)

• Optimization problem:

 $\underset{\mathbf{D}_{h},\mathbf{D}_{\ell},\mathbf{W}_{h},\mathbf{W}_{\ell}}{\operatorname{argmin}} \|\mathbf{D}_{h}\mathbf{W}_{h} - \mathbf{S}_{h}\|_{F} + \|\mathbf{D}_{\ell}\mathbf{W}_{\ell} - \mathbf{S}_{\ell}\|_{F} + \lambda_{h}\|\mathbf{W}_{h}\|_{1} + \lambda_{\ell}\|\mathbf{W}_{\ell}\|_{1},$

s.t. $\mathbf{W}_h = \mathbf{W}_\ell$, $||\mathbf{D}_h(:,i)||_2 \le 1$, $||\mathbf{D}_\ell(:,i)||_2 \le 1$

 $\Box \text{ Setting:} \mathbf{P} = \mathbf{D}_h \text{ and } \mathbf{Q} = \mathbf{D}_\ell$ $\min_{\mathbf{D}_h, \mathbf{W}_h, \mathbf{D}_l, \mathbf{W}_l} ||\mathbf{S}_h - \mathbf{D}_h \mathbf{W}_h||_F^2 + ||\mathbf{S}_l - \mathbf{D}_l \mathbf{W}_l||_F^2 + \lambda_l ||\mathbf{Q}||_1 + \lambda_h ||\mathbf{P}||_1$ s. t. $\mathbf{P} = \mathbf{W}_h, \mathbf{Q} = \mathbf{W}_l, \mathbf{W}_h = \mathbf{W}_l, ||\mathbf{D}_h(:, i)||_2 \le 1, ||\mathbf{D}_l(:, i)||_2 \le 1$

Augmented Lagrangian Function:

$$L(\mathbf{D}_{h}, \mathbf{D}_{l}, \mathbf{W}_{h}, \mathbf{W}_{l}, \mathbf{P}, \mathbf{Q}, Y_{1}, Y_{2}, Y_{3}) = \frac{1}{2} ||\mathbf{D}_{h}\mathbf{W}_{h} - \mathbf{S}_{h}||_{F}^{2} + \frac{1}{2} ||\mathbf{D}_{\ell}\mathbf{W}_{\ell} - \mathbf{S}_{\ell}||_{F}^{2} + \lambda_{h} ||\mathbf{P}||_{1} + \lambda_{\ell}||\mathbf{Q}||_{1} + \langle Y_{1}, \mathbf{P} - \mathbf{W}_{h} \rangle + \langle Y_{2}, \mathbf{Q} - \mathbf{W}_{\ell} \rangle + \langle Y_{3}, \mathbf{W}_{h} - \mathbf{W}_{\ell} \rangle + \frac{c_{1}}{2} ||\mathbf{P} - \mathbf{W}_{h}||_{F}^{2} + \frac{c_{2}}{2} ||\mathbf{Q} - \mathbf{W}_{\ell}||_{F}^{2} + \frac{c_{3}}{2} ||\mathbf{W}_{h} - \mathbf{W}_{\ell}||_{F}^{2}$$



ADMM for CDL - Algorithm

- Input: Training examples S_h and S_l, numb. of iterations: K and step size params. c1, c2, c3.
- 2. Initialization:
 - Dictionaries \rightarrow random selection of the columns of S_h and S_l
 - Lagrangian matrices $\rightarrow \mathbf{Y}_1 = \mathbf{Y}_2 = \mathbf{Y}_3 = 0$.
- 3. for k = 1,...,K do

$$\succ \quad \text{Update } \mathbf{W}_{h} \text{ and } \mathbf{W}_{l} \quad - \left[\begin{array}{c} \mathbf{W}_{h} = (\mathbf{D}_{h}^{T} \cdot \mathbf{D}_{h} + c_{1} \cdot I + c_{3} \cdot I)^{-1} \cdot (\mathbf{D}_{h}^{T} \cdot \mathbf{S}_{h} + Y_{1} - Y_{3} + c_{1} \cdot \mathbf{P} + c_{3} \cdot \mathbf{W}_{l}) \\ \mathbf{W}_{l} = (\mathbf{D}_{l}^{T} \cdot \mathbf{D}_{l} + c_{2} \cdot I + c_{3} \cdot I)^{-1} \cdot (\mathbf{D}_{l}^{T} \cdot \mathbf{S}_{l} + Y_{2} + Y_{3} + c_{2} \cdot \mathbf{Q} + c_{3} \cdot \mathbf{W}_{h}) \end{array} \right]$$

► Update **P** and **Q** → **P** =
$$S_{\lambda_h}\left(\left|\mathbf{W}_h - \frac{Y_1}{c_1}\right|\right)$$
 and: $\mathbf{Q} = S_{\lambda_l}\left(\left|\mathbf{W}_l - \frac{Y_2}{c_2}\right|\right)$

for
$$\mathbf{j} = \mathbf{1},...,\mathbf{N}$$
 do
> Update $\mathbf{\phi}_{\mathsf{h}}$ and $\mathbf{\phi}_{\mathsf{l}} \rightarrow \phi_{h} = \mathbf{W}_{h}(j,:) \cdot \mathbf{W}_{h}(j,:)^{T}$ and $\phi_{l} = \mathbf{W}_{l}(j,:) \cdot \mathbf{W}_{l}(j,:)^{T}$
> Update \mathbf{D}_{h} and \mathbf{D}_{l}
 $\mathbf{D}_{h}^{(k+1)}(:,j) = \mathbf{D}_{h}(:,j)^{(k)}(:,j) + \frac{\mathbf{S}_{h} \cdot \mathbf{W}_{h}(j,:)}{\phi_{h} + \delta}$
 $\mathbf{D}_{l}^{(k+1)}(:,j) = \mathbf{D}_{l}(:,j)^{(k)}(:,j) + \frac{\mathbf{S}_{l} \cdot \mathbf{W}_{l}(j,:)}{\phi_{l} + \delta}$

end

- **D** Normalize D_h and D_l between [0,1]
- **D Update** Lagrange multiplier matrices Y_1 , Y_2 and Y_3

end

Application: Spectral-Super Resolution

<u>**Task:**</u> Given few acquired spectral observations of a hyperspectral scene, synthesize the full spectrum



So far...

Hardware solutions

- Modify hyperspectral sensor's characteristics
- Additional optical elements

Key intuition!

- SSR → Post acquisition technique
- Inverse Imaging Problem
 - **Prior knowledge:** Sparse signal modeling

K. Fotiadou, G. Tsagkatakis, and P. Tsakalides: "Spectral Super-Resolution via Coupled Dictionary Learning." Submitted in IEEE Transactions Special Issue on Computational Imaging for Earth Sciences.



Proposed Spectral Coupled Dictionary Learning (SCDL) scheme





SSR Experimental Setup

Dictionaries Generation:

•100K coupled pairs of high & low-spectral resolution hyperspectral images

Hyperspectral Acquisition:

- **1. IMEC's snapshot mosaic hyperspectral instruments**
 - 5x5 snapshot mosaic hyperspectral sensor
 - Spectral resolution: 25 spectral bands
 - (600 875 nm)



Down-sampling factors : (x2), (x3), (x4)

> Evaluation Metrics: Peak signal to noise ratio (PSNR) $PSNR = 10log_{10}[L_{max}^2/MSE(x, y, \lambda)]$



Simulation Results: FORTH Building (x2)



Ground Truth, 636.57 nm



Ground Truth, 684.73 nm



Ground Truth, 724.24 nm



(x2) SCDL, **636.57** nm, SSIM: 0.97 DEDALE Tutorial Day Paris, November 2016



(x2) SCDL, **684.73** nm, SSIM: 0.96



(x2) SCDL, **724.24** nm, SSIM: 0.94

FORTH Building: Comparison with the state-of-the-art



(x2) Cubic, PSNR: 26.41 dB







Hyperion Data Recovery (x4 - zoom)



20th Band, x4, SSIM: 0.99



20th Band, Ground Truth



54th Band, *x4*, SSIM: 0.98



54th Band, Ground Truth

Parameters Full Spectrum ✓ Input: 67 spectral bands from the VNIR region (437 – 833 nm) ✓ Sub-sampling factor : x4 ✓ High Res. Dictionary: 67 bands ✓ Low Res. Dictionary: 17

PSNR Recovery of the 3D-cube:

<u>SCDL-SSR</u> \rightarrow 46.8 db





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Feature Learning



• Limitation: What is the optimal feature for each application?



Deep Learning: Big Picture

• **Challenge**: How could an artificial vision system learn appropriate internal representations automatically, the way humans seem to by simply looking at the world?





Why using deep learning?

• Traditional ("shallow") architectures



• Vs.... "deep" architectures



Advantage: Learn a *feature hierarchy* all the way from pixels to classifier!



Background: Typical Neural Networks





Motivation: Convolutional Neural Networks

- Limitations:
 - Full connectivity of traditional neural nets \rightarrow wasteful!
 - Tremendous number of parameters \rightarrow over-fitting!
 - − Example: 1000 x 1000 image → 1B parameters!!!





Convolutional Neural Networks (CNN's)

• Advantages

- Translation invariance
- Tied filter weights



Question: How to detect the accurate position of the eye?



- Answer: By pooling (max or average) filter responses at different locations
- ✓ robustness to the exact spatial location of the feature!



Convolutional Neural Networks (CNN's)



Feature maps

Application: ImageNet -2010 Contest^[1]

- □ 1.2 million high-resolution images
- □ 1,000 different classes
- □ 50,000 validation images, 150,000 testing images
- Top-1 error 47.1% best in contest, 45.7% best published
- Top-5 errors 28.2% best in contest, 25.7% best published



[1] Krizhevsky, Alex, Ilya Sutskever, and Geoffrey E. Hinton. "Imagenet classification with deep convolutional neural networks." *Advances in neural information processing systems*. 2012.



Application: Image Super-Resolution

Traditional Sparse-based Super-Resolution based on coupled trained dictionaries



J. Yang et al. "Image Super-Resolution via Sparse Representations". In IEEE transactions on image processing, 2010



Application: Image Super-Resolution^[1]



[1] C. Dong, et al. "Image Super-Resolution Using Deep Convolutional Networks", ECCV 2014



Relationship to the sparse coding based methods

Sparse Coding

- Extract LR patch & project on a LR dictionary, of size n₁
- Sparse coding solver, transform to HR sparse code, of size n₂

Project to HR dictionary,

SRCNN

- Apply n₁ linear filters on the input image
- Non linear mapping
- Linear convolution on the n₂ feature maps



Training Parameters (1/2)

Training:

- Small dataset: composed of 91 images
- Large Dataset: ~ 395.909 images (ImageNet!)
- **Testing:**
 - Set 5-5 images
 - Set 14-14 images
 - ImageNet

□ Scaling factors: x3

□ The more training data the better!







Training Parameters (2/2)

Sensitivity effects

Filter size







The deeper the better?
Sensitive to initialization params &

Larger filter size \rightarrow

Trade-off between

performance & speed

better results

learning rate



Application: Classification of HSI images



Preliminary Results

Experimental Setup

- 10 categories of indoors hyperspectral scenes
- Training Phase:
 - ▶ Pre-trained CNN model → AlexNet [1], [2]
 - CNN architecture: 23 layers

Testing Phase:

- Split the sets into training & validation
- Pick randomly 30% for training & 70% for testing
- Extract training features using CNN
- Train a Multiclass SVM Classifier using CNN features
- Evaluation of the Classifier

bag Classifier Bag → Correct !



[2] Vedaldi, Andrea, and Karel Lenc. "MatConvNet-convolutional neural networks for MATLAB." arXiv preprint arXiv:1412.4564 (2014).

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Proposed Mean Accuracy: 89%



Conclusions

□ Linear vs. non – linear sparse representations

- Widely used in multiple applications in signal / image processing
- □ Key component of sparse coding → the design of the proper dictionary
 - Single vs. Coupled Dictionary Learning
 - ADMM decomposition for coupled dictionary learning

Feature Learning

- Handcrafted vs. learnt feature operators
- Deep Learning
- □ What is the current trend in feature learning ?
 - Convolutional Neural Networks
 - ✓ **Applications** : Classification, Detection, Image restoration

